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Phenomenology of the General NMSSM with Gauge Mediated Supersymmetry Breaking

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Abstract

We investigate various classes of Gauge Mediated Supersymmetry Breaking models and show that the Next-to-Minimal Supersymmetric Standard Model can solve the μ-problem in a phenomenologically acceptable way. These models include scenarios with singlet tadpole terms, which are phenomenologically viable, e.g., in the presence of a small Yukawa coupling $\lesssim 10^{-5}$. Scenarios with suppressed trilinear A-terms at the messenger scale lead naturally to light CP-odd scalars, which play the rôle of pseudo R-axions. A wide range of parameters of such models satisfies LEP constraints, with CP-even Higgs scalars below 114 GeV decaying dominantly into a pair of CP-odd scalars.

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1 Introduction

The mediation of supersymmetry breaking to the observable sector via supersymmetric gauge interactions (GMSB) has already been proposed during the very early days of supersymmetric model building [1,2]. The essential ingredients of this class of models are a sequestered sector containing a spurion or a dynamical superfield $\widehat{X}$, whose F-component F_X does not vanish (there could exist several such fields). In addition, a messenger sector $\widehat{\varphi}_i$ exists, whose fields have a supersymmetric mass M, but a mass splitting between its scalar/pseudoscalar components due to its coupling to F_X. They carry Standard Model gauge quantum numbers such that the messengers couple to the Standard Model gauge supermultiplets. Possible origins of supersymmetry breaking in the form of a nonvanishing F_X component can be O'Raifeartaigh-type models [2], models based on no-scale supergravity [3,4] or Dynamical Supersymmetry Breaking [5–7].

If supersymmetric gauge interactions would be the only interactions that couple the visible sector with the messenger/sequestered sector, the phenomenologically required μ and $B\mu$ terms of the Minimal Supersymmetric Standard Model (MSSM) would be difficult to generate. The simplest solution to this problem is the introduction of a gauge singlet superfield $\widehat{S}$ and a superpotential including the $\lambda \widehat{S} \widehat{H}_u \widehat{H}_d$ term, which has been used in early globally [8] and locally supersymmetric [9] models.

Let us point out a possible connection between gravity mediated supersymmetry breaking and GMSB-like models [3,4]: standard gravity mediated supersymmetry breaking within the MSSM requires Giudice-Masiero-like terms (depending on the Higgs doublets) in the Kähler potential [10] in order to generate the μ and $B\mu$ terms (see [11] for a possible 5-dimensional origin of such terms). Given a possible source for such terms, one can replace the Higgs doublets by the messengers of GMSB models and proceed as in the usual analysis of gauge mediation. The advantage of such models is that *no other* gravity mediated source of supersymmetry breaking as scalar or gaugino soft masses is required; such sources of supersymmetry breaking are frequently absent in higher dimensional setups. On the other hand, the solution of the standard μ-problem for the Higgs doublets still requires the introduction of a singlet $\widehat{S}$. Then one is also led to the scenario considered in this paper, the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with gauge mediated supersymmetry breaking.

In order to generate a sufficiently large vacuum expectation value of the scalar component S of $\widehat{S}$ (and hence a sufficiently large effective μ term $\mu_{eff} = \lambda \langle S \rangle$), the singlet superfield $\widehat{S}$ should possess additional Yukawa interactions with the messenger/sequestered sector. Then, an effective potential for S with the desired properties can be radiatively generated.

Note that the so-called singlet tadpole problem [12] is absent once the original source of supersymmetry breaking is of the F-type [3,4,13]. On the contrary, singlet tadpole diagrams can now generate the desired structure of the singlet effective potential [3,4], triggering a VEV of S. If the singlet couples at lowest possible loop order to the messenger/sequestered sector such that tadpole diagrams are allowed, a mild version of the singlet tadpole problem reappears, since the coefficients of the corresponding terms linear in S are typically too large. This milder problem can be solved under the assumption that the involved Yukawa coupling is sufficiently small – however, it does not need to be smaller than the electron Yukawa

1

coupling of the Standard Model (see below).

In the meantime, quite a large number of models involving GMSB and at least one gauge singlet, that generates an effective μ term, have been studied [14–19]. They differ in the particle content of the messenger/sequestered sector, and include sometimes more than one gauge singlet superfield.

The purpose of the present paper is the investigation of a large class of models obtained after integrating out the messenger/sequestered sector (including possibly heavy singlet fields). It is assumed that the remaining particle content with masses below the messenger scale M is the one of the NMSSM.

The couplings and mass terms of the NMSSM are obtained under the following assumptions:

– no interactions between the Higgs doublets H_u, H_d and the messenger/sequestered sector exist apart from supersymmetric gauge interactions; then no MSSM-like μ or $B\mu$ terms are generated after integrating out the messenger/sequestered sector;

– the gauge singlet superfield $\widehat{S}$ has Yukawa interactions with the messenger/sequestered sector. As a result, various soft terms and $\widehat{S}$-dependent terms in the superpotential can be generated after integrating out the messenger/sequestered sector.

Under the only assumption that the original source of supersymmetry breaking is F_X and that the messengers have a mass of the order $M \gtrsim \sqrt{F_X}$, superspace power counting rules allow to estimate the maximally possible order of magnitude of the generated masses and couplings.

In general, these masses and couplings will comprise nearly all possibilities consistent with gauge invariance (see Section 2), leading to the general NMSSM. However, many of these mass terms and couplings can be much smaller, or absent, than indicated by the power counting rules (but never larger), if the corresponding diagrams involve high loop orders, small Yukawa couplings, or are forbidden by discrete or (approximate) continuous symmetries.

In the next Section, we will parametrize the mass terms and couplings of the general NMSSM, and estimate their (maximally possible) radiatively generated order of magnitude with the help of superspace power counting rules. Section 3 is devoted to a phenomenological analysis of three different scenarios, which are defined by particular boundary conditions for the NMSSM parameters at the messenger scale, and Section 4 contains our conclusions.

2   Results of superspace power counting rules

The class of models investigated in this paper is defined by a superpotential

$$W = \lambda \widehat{S} \widehat{H}_u \widehat{H}_d + \frac{\kappa}{3} \widehat{S}^3 + \widetilde{W}(\widehat{S}, \widehat{X}, \widehat{\varphi}_i, \ldots) + \ldots , \qquad (2.1)$$

where $\widetilde{W}(\widehat{S}, \widehat{X}, \widehat{\varphi}_i, \ldots)$ denotes the couplings of $\widehat{S}$ to the messenger/sequestered sector, and we have omitted the standard Yukawa couplings of $\widehat{H}_u$ and $\widehat{H}_d$. No MSSM-like μ-term is

assumed to be present. Due to a coupling $\widehat{X}\widehat{\varphi}_i\widehat{\varphi}_i$ in $\widetilde{W}$, a non-vanishing F_X-component

$$F_X = m^2 \tag{2.2}$$

induces a mass term

$$\frac{1}{2}m^2\left(A_{\varphi_i}^2 + A_{\varphi_i}^{*\,2}\right) \tag{2.3}$$

which gives opposite contributions to the squared masses of the real and imaginary components of the scalar components of the messengers $\widehat{\varphi}_i$. Since we assume no direct couplings of $\widehat{S}$ to $\widehat{X}$, this constitutes the only original source of supersymmetry breaking.

After integrating out the messenger/sequestered sector, the remaining effective action for the light superfields $\widehat{\Phi}$ (the fields $\widehat{S}, \widehat{H}_u, \widehat{H}_d, \ldots$ of the NMSSM) is necessarily of the form

$$\sum_i c_i \int d^4\theta f_i(D_\alpha, \overline{D}_{\dot{\alpha}}, \widehat{\Phi}, \overline{\widehat{\Phi}}, \widehat{X}, \overline{\widehat{X}}) \ , \tag{2.4}$$

where the relevant terms are obtained after the replacement of at least one superfield $\widehat{X}$ by its F-component F_X. The maximally possible orders of magnitude of the coefficients c_i can be obtained by dimensional analysis: if a function f_i is of a mass dimension $[M]^{d_f}$, the corresponding coefficient c_i has a mass dimension $[M]^{2-d_f}$. As long as $d_f \geq 2$ (which will be the case), c_i will typically depend on the mass of the heaviest particle running in the loops to the appropriate power, and subsequently we identify this mass M with a unique messenger scale M_{mess}.

We are aware of the fact that models exist where the c_i depend on several mass scales M_i; however, it is always trivial to identify a mass scale M such that c_i are bounded from above by M^{2-d_f}. Also, in the particular case $d_f = 2$, c_i can involve large logarithms; these depend on whether the VEV F_X is "hard" (i.e. generated at a scale Λ much larger than M) or "soft", i.e. generated by a potential involving terms of the order of M. In the first case, logarithms of the form $\ln(\Lambda^2/M^2)$ can appear in c_i.

In the present situation (no interactions between the Higgs doublets H_u, H_d and the messenger/sequestered sector) possible supercovariant derivatives $D_\alpha, \overline{D}_{\dot{\alpha}}$ inside f_i do not lead to terms that would otherwise be absent; for this reason we will omit them in our analysis. (Here, we will not discuss the radiatively generated gaugino masses and scalar masses for the gauge non-singlets, but concentrate on the NMSSM specific effects.) To lowest loop order we can use the underlying assumption that only the singlet superfield $\widehat{S}$ has direct couplings to the messenger/sequestered sector (however, see Fig. 1 below). The first terms that we will investigate are then of the form

$$\sum_i c_i \int d^4\theta f_i(\widehat{S}, \overline{\widehat{S}}, \widehat{X}, \overline{\widehat{X}}) \ . \tag{2.5}$$

Below we list all relevant terms with this structure. Given an expression of the form (2.5), the generated S- and F_S-dependent terms can be obtained by the replacements

$$\widehat{X} = M + \theta^2 m^2, \qquad \widehat{S} = S + \theta^2 F_S \ . \tag{2.6}$$

3

Due to the coupling $\widehat{X}\widehat{\varphi}_i\widehat{\overline{\varphi}}_i$, the supersymmetry conserving mass M of the messengers $\widehat{\varphi}_i$ can be identified with the value of the scalar component of $\widehat{X}$. Loop factors like $(16\pi^2)^{-1}$ and model dependent Yukawa couplings are not explicitly given, but we indicate the powers of m (which follow from the powers of F_X) and M (which follow from dimensional analysis).

The possible operators f_i and the corresponding contributions to the scalar potential are then given by:

$$\widehat{S}\overline{\widehat{X}} + h.c. : \quad m^2 F_S + h.c. \tag{2.7}$$

$$\widehat{S}\overline{\widehat{X}}\widehat{X} + h.c. : \quad \frac{m^4}{M}S + m^2 F_S + h.c. \tag{2.8}$$

$$\widehat{S}\overline{\widehat{S}}\widehat{X} + h.c. : \quad \frac{m^2}{M}(SF_S^* + h.c.) + F_S F_S^* \tag{2.9}$$

$$\widehat{S}\overline{\widehat{S}}\widehat{X}\overline{\widehat{X}} : \quad \frac{m^4}{M^2}SS^* + \frac{m^2}{M}(SF_S^* + h.c.) + F_S F_S^* \tag{2.10}$$

$$\widehat{S}\widehat{S}\overline{\widehat{X}} + h.c. : \quad \frac{m^2}{M}(SF_S + h.c.) \tag{2.11}$$

$$\widehat{S}\widehat{S}\widehat{X}\overline{\widehat{X}} + h.c. : \quad \frac{m^4}{M^2}S^2 + \frac{m^2}{M}SF_S + h.c. \tag{2.12}$$

Operators with higher powers of $\widehat{X}$ or $\overline{\widehat{X}}$ do not generate new expressions, and operators with higher powers of $\widehat{S}$ or $\overline{\widehat{S}}$ generate negligible contributions with higher powers of M in the denominator (recall that we are assuming $M \gtrsim m$).

The terms $\sim F_S F_S^*$ in (2.9) and (2.10) only account for a correction to the wave function normalization of the superfield $\widehat{S}$, which can be absorbed by a redefinition of $\widehat{S}$. The remaining terms can be written as an effective superpotential ΔW and additional contributions ΔV_{soft} to the soft terms of the general NMSSM. To this end, the terms $SF_S^* + h.c.$ in (2.9) and (2.10) have to be rewritten using the expression derived from the superpotential (2.1):

$$F_S^* = \lambda H_u H_d + \kappa S^2 + \ldots \tag{2.13}$$

where the dots stand for terms of higher order in the loop expansion. We parametrize the effective superpotential ΔW and the soft terms ΔV_{soft} of the general NMSSM in agreement with SLHA2 conventions [20]:

$$\Delta W = \mu'\widehat{S}^2 + \xi_F\widehat{S} , \tag{2.14}$$

$$\Delta V_{soft} = m_S^2|S|^2 + (\lambda A_\lambda SH_uH_d + \frac{1}{3}\kappa A_\kappa S^3 + m_S'^2 S^2 + \xi_S S + h.c.) . \tag{2.15}$$

Then the expressions (2.7) to (2.12) lead to

$$\mu' \sim \frac{m^2}{M} , \tag{2.16}$$

$$\xi_F \sim m^2 , \tag{2.17}$$

$$m_S^2 \sim \frac{m^4}{M^2} , \tag{2.18}$$

$$A_\lambda = \frac{1}{3} A_\kappa \quad \sim \quad \frac{m^2}{M} \ , \tag{2.19}$$

$$m_S'^2 \quad \sim \quad \frac{m^4}{M^2} \ , \tag{2.20}$$

$$\xi_S \quad \sim \quad \frac{m^4}{M} \ . \tag{2.21}$$

Next, within the class of models defined by the superpotential (2.1), there exist the diagrams shown in Fig.1 which generate terms in ΔV_{soft} which are not included in the list (2.16) – (2.21). The corresponding operators and soft terms (after the replacement of F_{H_u} and F_{H_d} by their tree level expressions) are given by

$$\widehat{H}_u \overline{\widehat{H}}_u \widehat{X} + h.c. \quad \rightarrow \tfrac{m^2}{M} H_u F_{H_u}^* \quad \rightarrow \Delta A_\lambda = \Delta A_t \sim \frac{m^2}{M} \ , \tag{2.22}$$

$$\widehat{H}_u \overline{\widehat{H}}_u \widehat{X} \overline{\widehat{X}} \quad \rightarrow \tfrac{m^4}{M^2} H_u H_u^* \quad \rightarrow \Delta m_u^2 \sim \frac{m^4}{M^2} \ , \tag{2.23}$$

together with analogous expressions with H_u replaced by H_d (and A_t by A_b).
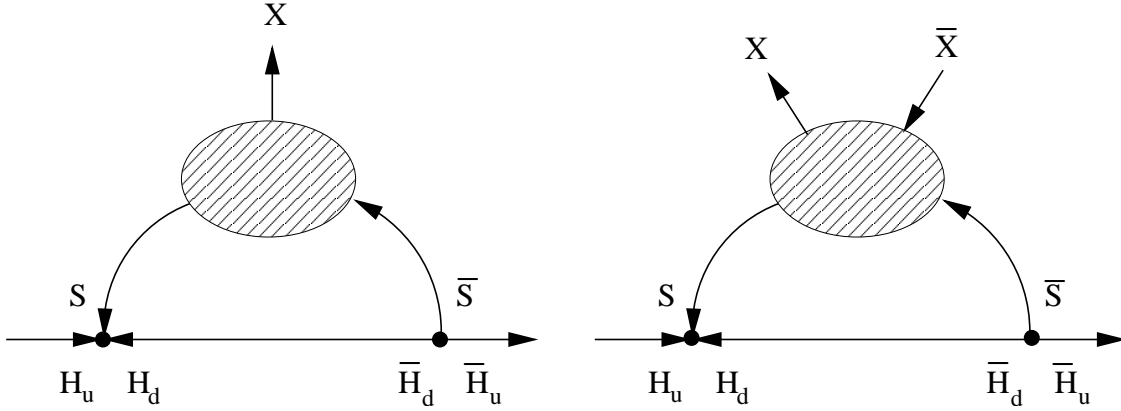

Figure 1: Superfield diagrams which generate the operators (2.22) and (2.23) (omitting, for simplicity, the "hats" on top of the letters denoting the superfields.)

Similar expressions are also generated by i) the replacement of the shaded bubbles in Fig.1 by the effective operators (2.9) and (2.10) (which generate the soft terms (2.18) and (2.17)), and ii) the Renormalization Group (RG) evolution of A_λ, A_t, A_b, m_u^2 and m_d^2 from the messenger scale M down to the weak (or SUSY) scale M_{SUSY}. Whereas this RG evolution sums up potentially large logarithms of the form $\ln(M^2/M_{SUSY}^2)$, it does not describe contributions without such logarithms which serve as boundary conditions for the RG evolution at the scale $Q^2 = M^2$.

Note that both contributions (2.22) and (2.23) are generated only at (or beyond) two loop order, and are hence suppressed by additional factors $\lambda^2/(16\pi^2)^2 \times$ additional Yukawa

couplings. Compared to the effective SUSY breaking scale $m^2/(16\pi^2 M)$, the contribution to the A terms (2.22) is negligibly small. However, the contribution (2.23) to $\Delta m_u^2 = \Delta m_d^2$ can be of the same order as the two loop contributions mediated by gauge interactions (see appendix A), if λ is not too small. Since the contribution (2.23) to $\Delta m_u^2 = \Delta m_d^2$ is typically negative, we will subsequently parametrize it in terms of Δ_H defined as

$$\Delta m_u^2 = \Delta m_d^2 = -\Delta_H \frac{\lambda^2}{(16\pi^2)^2} M_{SUSY}^2 \qquad (2.24)$$

with $M_{SUSY} = m^2/M$ as in appendix A, and Δ_H bounded from above by $\Delta_H \lesssim$ (Yukawa)^2 $\lesssim \mathcal{O}(1)$.

To summarize this Section, within the class of models defined by the superpotential (2.1) one obtains in general, after integrating out the messenger/sequestered sector, an effective NMSSM valid at scales below the messenger scale M, which includes
a) the first two terms in the superpotential (2.1),
b) the soft SUSY breaking gaugino, squark, slepton and Higgs masses obtained by gauge mediation, which we recall for convenience in appendix A,
c) additional terms in the superpotential (2.14) and additional soft terms (2.15),
d) additional contributions to the soft SUSY breaking Higgs masses as in (2.24).

Note that neither an explicit μ term nor an explicit $m_3^2 \equiv B\mu$ term are present at the messenger scale M. However, once the above soft terms are used as boundary conditions for the RG evolution from M down to M_{SUSY}, a term of the form $m_3^2 H_u H_d$ can be radiatively generated in general. (In the appendix B, we recall the β-functions of the parameters of the Higgs sector of the general NMSSM. One finds that a non-vanishing parameter $m_S'^2$ generally induces a non-vanishing m_3^2.)

Depending on the structure of the messenger/sequestered sector, many of the terms in (2.16) – (2.21) can be disallowed or suppressed by discrete or approximate continuous symmetries. (Exact continuous symmetries forbidding any of these terms would be spontaneously broken in the physical vacuum, giving rise to an unacceptable Goldstone boson.) An exception is the term (2.10) leading to the soft singlet mass term (2.18), which can never be suppressed using symmetries. However, precisely this term is often generated only to higher loop order and/or to higher order in an expansion in m/M as expected from naïve dimensional analysis [4, 18]. Finally we remark that terms of the form $SF_S^* + h.c.$ (which give rise to the trilinear soft terms (2.19)) will be suppressed if an R-symmetry is only weakly broken in the scalar sector.

3 Phenomenological analysis

The purpose of this Section is the phenomenological analysis of various scenarios within the class of models defined in Section 2, that differ by the presence/absence of the different terms (2.16) to (2.21) and (2.24).

To this end we employ a Fortran routine NMGMSB, that will be made public on the NMSSMTools web page [21]. The routine NMGMSB is a suitable generalization of the

routine NMSPEC (available on the same web site) towards the general NMSSM with soft SUSY breaking terms specified by GMSB, i.e. it allows for a phenomenological analysis of the class of models defined in Section 2. It requires the definition of a model in terms of the parameter λ and the soft SUSY breaking and superpotential terms b) – d) above. Since the coupling λ at the effective SUSY breaking scale plays an important phenomenological rôle (and in order to allow for comparisons with other versions of the NMSSM as mSUGRA inspired), the coupling λ on input is defined at an effective SUSY breaking scale Q_{SUSY} given essentially by the squark masses. The remaining input parameters, notably the soft SUSY breaking terms listed in appendix A and in (2.16) – (2.21), are defined at a unique messenger scale M.

The RG equations are then integrated numerically from M down to Q_{SUSY}. Additional input parameters are, of course, M_Z, and also $\tan\beta$ (at the scale M_Z). Similar to the procedure employed in NMSPEC, the minimization equations of the effective Higgs potential – including radiative corrections as in [21] – can then be solved for the Yukawa coupling κ in the superpotential (2.1), and for the SUSY breaking singlet mass m_S^2 (2.18) or, if m_S^2 is fixed as input, for ξ_S. (If specific values for κ, m_S^2 and ξ_S at the scale M are desired as input, this procedure is somewhat inconvenient. Then, one would have to scan over at least some of the other input parameters and select points in parameter space where κ, m_S^2 or ξ_S – which are given at the scale M as output – are close enough to the desired numerical values.) Since the gauge and SM Yukawa couplings are defined at the scale M_Z, a few iterations are required until the desired boundary conditions at M_Z and M are simultaneously satisfied.

After checking theoretical constraints as the absence of deeper minima of the effective potential and Landau singularities below M, the routine proceeds with the evaluation of the physical Higgs masses and couplings (including radiative corrections as in [21]) and the sparticle spectrum including pole mass corrections. Then, phenomenological constraints can be checked:
– Higgs masses, couplings and branching ratios are compared to constraints from LEP, including constraints on unconventional Higgs decay modes [22] relevant for the NMSSM;
– constraints from B-physics are applied as in [23], and the muon anomalous magnetic moment is computed.

Subsequently we investigate several scenarios, for which many (but different) terms in the list (2.16) – (2.21) vanish or are negligibly small.

3.1   Scenarios with tadpole terms

The tadpole terms ξ_F in ΔW in (2.14) and ξ_S in ΔV_{soft} in (2.15) will trigger a nonvanishing VEV of S. However, as it becomes clear from (2.17) and (2.21), these tadpole terms – if not forbidden by symmetries – tend to be too large: the scale of the soft SUSY breaking gaugino, squark, slepton and Higgs masses in GMSB models is given by $M_{SUSY} \sim m^2/M$ (together with an additional loop factor $(16\pi^2)^{-1}$, see appendix A). Written in terms of M and M_{SUSY}, the maximally possible order of magnitude of the supersymmetric and soft SUSY breaking tadpole terms are $\xi_F \sim m^2 \sim MM_{SUSY}$ and $\xi_S \sim m^4/M \sim MM_{SUSY}^2$. If $M \gg M_{SUSY}$, which will generally be the case, these tend to be larger than the desired orders of magnitude

$\xi_F \sim M_{SUSY}^2$ and $\xi_S \sim M_{SUSY}^3$. (This problem is similar to the μ and $B\mu$ problem in the MSSM with GMSB, see [14].) Hence one has to assume that these terms are suppressed, e.g. generated to higher loop order only as in [3], or involve small Yukawa couplings. Let us study the latter scenario quantitatively in a simple model [4]: let us assume that the singlet superfield couples directly to n_5 pairs of messengers $\widehat{\phi}$, $\widehat{\overline{\phi}}$ (in $\underline{5}$ and $\underline{\overline{5}}$ representations under $SU(5)$) due to a term

$$- \eta \widehat{S} \widehat{\overline{\phi}} \widehat{\phi} \tag{3.25}$$

in the superpotential $\widetilde{W}$ in (2.1). Then, one loop diagrams generate [4]

$$\xi_F = n_5 \frac{\eta}{8\pi^2} m^2 \ln \left(\Lambda^2 / M^2 \right) \tag{3.26}$$

and

$$\xi_S = -n_5 \frac{\eta}{16\pi^2} \frac{m^4}{M} \tag{3.27}$$

in agreement with the power counting rules (2.17) and (2.21). (The UV cutoff Λ appears in (3.26) only if the SUSY breaking F_X is "hard" in the sense discussed in Section 2; otherwise the logarithm in (3.26) should be replaced by a number of $\mathcal{O}(1)$.)

Below, we consider a mass splitting $m^2 \sim 8 \times 10^{10}$ GeV^2 among the messenger scalars and pseudoscalars, and a messenger scale $M \sim 10^6$ GeV. Then, for $\ln \left(\Lambda^2 / M^2 \right) \sim 3$, a Yukawa coupling $\eta \sim 2 \times 10^{-6}$ generates $\xi_F \sim (150$ GeV$)^2$ and $\xi_S \sim -(1$ TeV$)^3$. We find that these orders of magnitude for ξ_F and ξ_S are perfectly consistent with a phenomenologically viable Higgs sector. Given the presence of small Yukawa couplings in the Standard Model, and the possibility of obtaining additional symmetries in the limit of vanishing η, we do not consider $\eta \sim 10^{-6} - 10^{-5}$ as particularly unnatural.

The coupling (3.25) also gives rise to a positive SUSY breaking mass squared

$$m_S^2 = n_5 \frac{\eta^2}{4\pi^2} \frac{m^6}{M^4} \tag{3.28}$$

for the singlet S. Under the assumption of such small values for η, this term is numerically negligible (as well as contributions to $A_\lambda, A_\kappa, \mu', m_S'^2, \Delta_H$ and two loop contributions to m_S^2 of $\mathcal{O}(m^4/M^2)$).

Hence in the following we will concentrate on models where, among the terms in (2.16) – (2.21) and (2.24), only ξ_F and ξ_S are nonvanishing. (These models are then similar to the ones denoted as "nMSSM" in [24]. However, given the present constraints on the soft terms we found that a term $\sim \kappa$ in the superpotential (2.1) is required for the stability of the scalar potential.) The remaining free parameters are $\tan\beta$, λ, M, m^2/M and ξ_F: since m_S^2 is fixed as input at the scale M (where $m_S^2 = 0$), the equation following from the minimization of the potential w.r.t. S can be used to determine ξ_S.

Quite generally, there exist two distinct allowed regions in the parameter space, which differ how the lightest scalar Higgs mass m_{h_1} satisfies the LEP bound of ~ 114 GeV:
a) region A at low $\tan\beta$ and large λ, where the NMSSM specific contributions to the lightest Higgs mass allow for values above above 114 GeV. Low values of $\tan\beta$ demand that the

8

messenger scale M is not too large: $\tan\beta \sim 1$ requires $m_u^2 \sim m_d^2$ at the SUSY scale, but the RG equation for m_u^2 differs from the one for m_d^2 by the presence of the top Yukawa coupling (which is particularly large for small $\tan\beta$). Thus the range of the RG running should not be too big, i.e. the scale M should not be too far above the SUSY scale.

b) region B at large $\tan\beta$, where the messenger scale M is quite large (typically $\sim 10^{13}$ GeV) resulting in stop masses in the 1.5–2 TeV range. Then the top/stop radiative corrections to the lightest Higgs mass can lift it above 114 GeV without the need for NMSSM specific contributions. (At large $\tan\beta$, λ does not increase the lightest Higgs mass; on the contrary, large values of λ lower its mass through an induced mixing with the singlet-like scalar. Hence, λ must be relatively small here.) However, in the present context one finds from (3.26) and (3.27) that such large values for M (with fixed $m^2/M \sim 10^5$ GeV) would require extremely small values for η. For this reason we confine ourselves to region A in the following.

In region A, the LEP bound on m_{h_1} requires $\tan\beta$ to be smaller than ~ 2, and λ larger than ~ 0.45. Subsequently we investigate the interval $0.45 < \lambda < 0.6$ and $\tan\beta > 1.2$, where perturbativity in the running Yukawa couplings λ, κ and h_t is guaranteed at least up to the messenger scale M. If we naïvely extrapolate the RGEs beyond the scale M (taking the contributions of the messenger fields to the running gauge couplings into account), perturbativity in the running Yukawa couplings is usually not satisfied up to the GUT scale in region A (in contrast to scenarios where $M \sim 10^{13}$ GeV). There exist different possible solutions to this problem: first, additional matter could be present at the messenger scale, charged under the SM gauge groups. Then, SM gauge couplings can become large (at the boundary of perturbativity) below the GUT scale, and since they induce a negative contribution to the β functions for h_t and λ, they could help to avoid a Landau singularity in the Yukawa sector below M_{GUT}. Another attitude would be to assume that a strongly interacting sector (possibly responsible for the breaking of supersymmetry) exists at or above the messenger scale M; then the singlet S, for example, could turn out to be a composite state which would imply a compositeness condition equivalent to Landau singularities in the Yukawa couplings of S at the corresponding scale (without affecting, at the one loop level, the grand unification of the SM gauge couplings).

Within the region $1.2 < \tan\beta < 2$ and $0.45 < \lambda < 0.6$, a wide range of the remaining parameters M, m^2/M and ξ_F satisfies all phenomenological constraints. Subsequently we fix these parameters near the center of the allowed range: $M = 10^6$ GeV, $m^2/M = 8 \times 10^4$ GeV and $\xi_F = 3 \times 10^4$ GeV^2, and vary $\tan\beta$ and λ in the above intervals (taking, for simplicity, $n_5 = 1$).

The allowed range of $\tan\beta$ ($\tan\beta < 1.6$ for these values for M, m^2/M and ξ_F) and λ (actually $\lambda \gtrsim 0.5$) is shown in Fig. 2; the upper limit on $\tan\beta$ originates from the LEP bound on the lightest Higgs mass m_{h_1}. This becomes evident from Fig. 3, where we show the range of m_{h_1} (for various values of λ, larger values of λ corresponding to larger values of m_{h_1}) as a function of $\tan\beta$. If we would allow for larger values of λ (and/or smaller values of $\tan\beta$), larger values for m_{h_1} would be possible.

In Fig. 4 we display the charged Higgs mass $m_{h^\pm}$ (practically degenerated with a scalar with mass m_{h_2} and a pseudoscalar with mass m_{a_2}), the singlet-like scalar mass m_{h_3} and the singlet-like pseudoscalar mass m_{a_1}, all of which are nearly independent of λ. For small
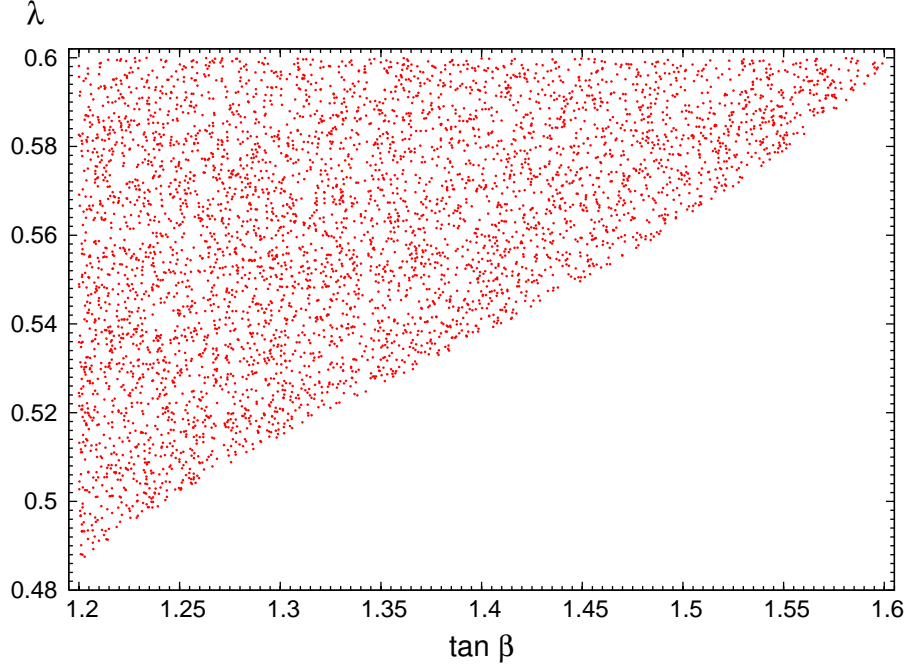
9

Figure 2: Allowed values of λ as a function of $\tan\beta$ for $M = 10^6$ GeV, $M_{SUSY} = m^2/M = 8 \times 10^4$ GeV and $\xi_F = 3 \times 10^4$ GeV^2.

$\tan\beta$ the large values of the Higgs masses indicate that this region is implicitly more fine tuned. The remaining sparticle spectrum is essentially specified by M and m^2/M, and hardly sensitive to $\tan\beta$ and λ within the above intervals:

$$
\begin{aligned}
\text{Bino}: &\quad \sim 105 \text{ GeV} \\
\text{Winos}: &\quad \sim 200 \text{ GeV} \\
\text{Higgsinos}: &\quad \sim 670 - 1000 \text{ GeV} \\
\text{Singlino}: &\quad \sim 900 - 1800 \text{ GeV} \\
\text{Sleptons}: &\quad \sim 140 - 290 \text{ GeV} \\
\text{Squarks}: &\quad \sim 640 - 890 \text{ GeV} \\
\text{Gluino}: &\quad \sim 660 \text{ GeV}
\end{aligned}
$$

(Due to the small value of $\tan\beta$ in this scenario, the supersymmetric contribution to the muon anomalous magnetic moment is actually too small to account for the presently observed deviation w.r.t. the Standard Model.)

In Fig. 5, we give the values of ξ_S (at the scale M), which are obtained as an output as function of $\tan\beta$. Within the model corresponding to (3.25) – (3.27) above, one can easily deduce the Yukawa coupling η from ξ_S using (3.27) resulting in η varying in the range 2×10^{-6} (for $\tan\beta = 1.6$) to 10^{-5} (for $\tan\beta = 1.2$). The corresponding value of $\ln(\Lambda^2/M^2)$ can then be deduced from (3.26), with the conclusion that $\ln(\Lambda^2/M^2)$ should assume values
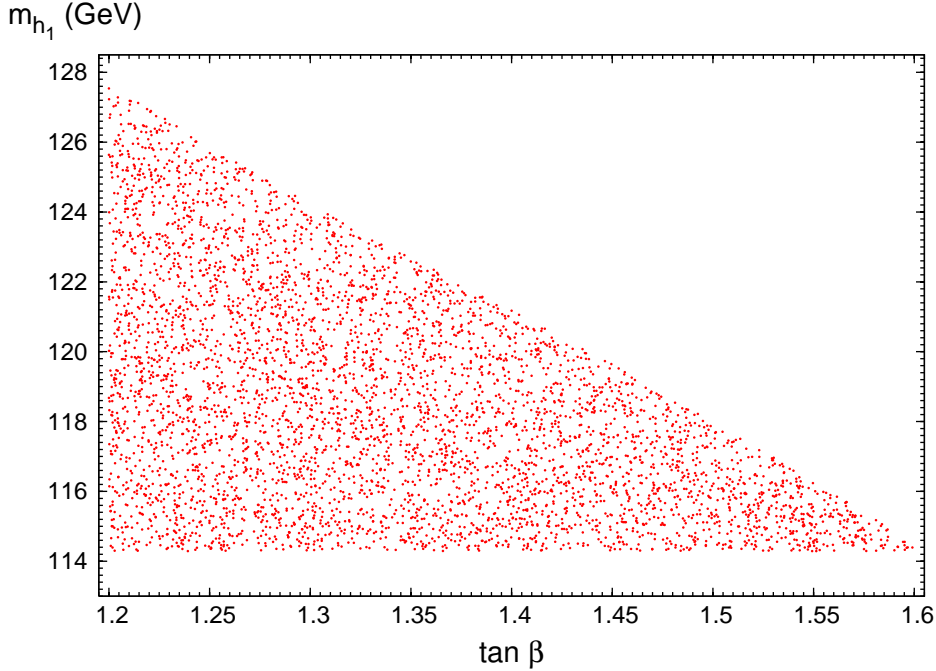
10

Figure 3: The lightest Higgs mass as a function of $\tan\beta$ for the same parameters as in Fig. 2, larger values of m_{h_1} corresponding to larger values of λ.

in the range 1 to 4 – a reasonable result, by no means guaranteed, that we consider as a strong argument in favour of such a simple model.

Finally we note that for larger values of n_5 (as $n_5 = 3$), M (as $M = 2 \times 10^{10}$) and ξ_F (as $\xi_F = 10^5$ GeV^2, see also the next subsection) phenomenologically viable regions in parameter space exist where the running Yukawa couplings λ, κ and h_t remain perturbative up to M_{GUT}. Within the model above, these scenarios would require an even smaller Yukawa coupling η, $\eta \sim 10^{-8}$.

3.2 Scenarios without tadpole terms

Scenarios without tadpole terms have been proposed in [16]. If the number of messengers is doubled ($n_5 = 2$), i.e. introducing $\widehat{\Phi}_1$, $\widehat{\overline{\Phi}}_1$, $\widehat{\Phi}_2$ and $\widehat{\overline{\Phi}}_1$, these can couple to $\widehat{S}$ and to the spurion $\widehat{X}$ in such a way that a discrete Z_3 symmetry is left unbroken by the VEV of $\widehat{X}$ [16]:

$$\widetilde{W} = \widehat{X}\left(\widehat{\overline{\Phi}}_1\widehat{\Phi}_1 + \widehat{\overline{\Phi}}_2\widehat{\Phi}_2\right) + \eta\widehat{S}\widehat{\overline{\Phi}}_1\widehat{\Phi}_2 \qquad (3.29)$$

Then tadpole terms $\sim \xi_F$ and $\sim \xi_S$ are disallowed, and the Yukawa coupling η can be much larger. These scenarios have been recently investigated in [18] (see also [19]), where the $SU(5)$ breaking (generated via the RG equations between M_{GUT} and M) inside $\eta\widehat{S}\widehat{\overline{\Phi}}_1\widehat{\Phi}_2$ has been taken into account.
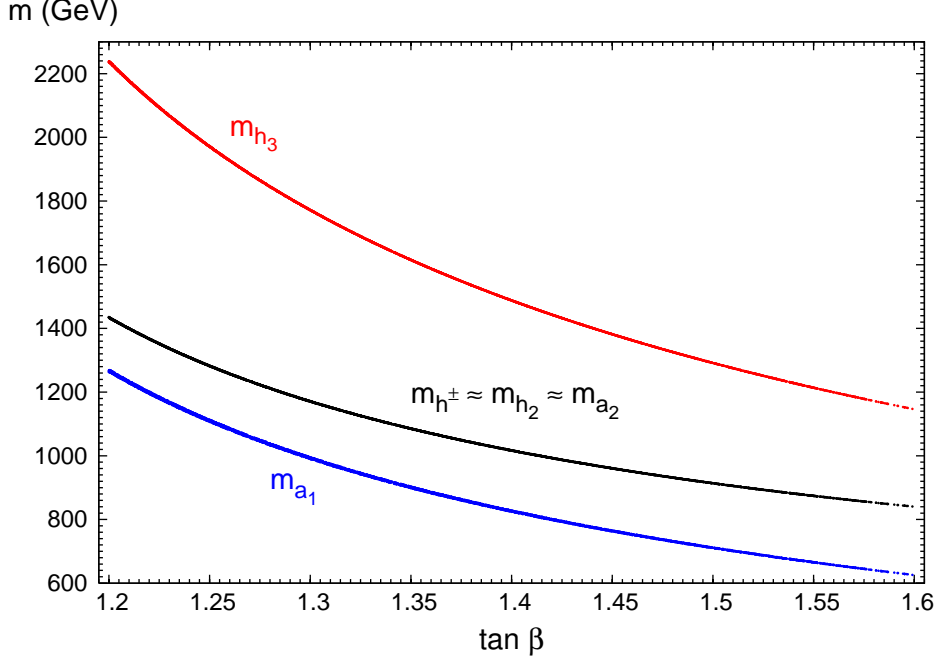
11

Figure 4: Heavy Higgs masses as a function of $\tan\beta$ for the same parameters as in Fig. 2.

For larger values of η, messenger loops generate non-negligible values for the singlet mass m_S^2 (2.18), trilinear A-terms (2.19) and corrections $\Delta m_u^2 = \Delta m_d^2$ as in (2.24) at the scale M [18]. Phenomenologically viable regions in parameter space have been found in [18], where the parameters M and M_{SUSY} have been chosen as $M = 10^{13}$ GeV and $M_{SUSY} = m^2/M = 1.72 \times 10^5$ GeV. The stop masses are quite large (up to ~ 2 TeV) such that the stop/top induced radiative corrections to m_{h_1} lift it above the LEP bound of ~ 114 GeV.

We have re-investigated this scenario in a somewhat simpler setup: first we observe that the generated values for A_κ and Δ_H, in the notation (2.19) and (2.24), are always related by

$$A_\kappa = -\frac{3}{16\pi^2}\Delta_H M_{SUSY} \tag{3.30}$$

(with $\Delta_H = 2\xi_D^2 + 3\xi_T^2$ in the notation of [18], where $\xi_{D,T}$ denote Yukawa couplings corresponding to our η in (3.29). At M_{GUT} one has $\xi_D = \xi_T \equiv \xi_U$ [18].) The singlet mass at the scale M is then of the order

$$m_S^2 \simeq \frac{1}{(16\pi^2)^2}\left(\frac{7}{5}\Delta_H^2 - \frac{1}{5}(16g_3^2 + 6g_2^2 + \frac{10}{3}g_1^2)\Delta_H - 4\kappa^2\Delta_H\right)M_{SUSY}^2 , \tag{3.31}$$

where we have neglected the $SU(5)$ breaking among the Yukawas at the scale M.

We tried to reproduce the three phenomenologically viable regions in parameter space studied in [18]: region I where $\xi_U \ll 1$, region III where $0.6 \lesssim \xi_U \lesssim 1.1$, and region II where $1.3 \lesssim \xi_U \lesssim 2$. We observe, however, that for $\xi_U \gtrsim 0.7$ (or $\Delta_H \gtrsim 1.5$ after taking the running of ξ_U between M_{GUT} and M into account) the generated value for $|A_\kappa|$ from (3.30) exceeds
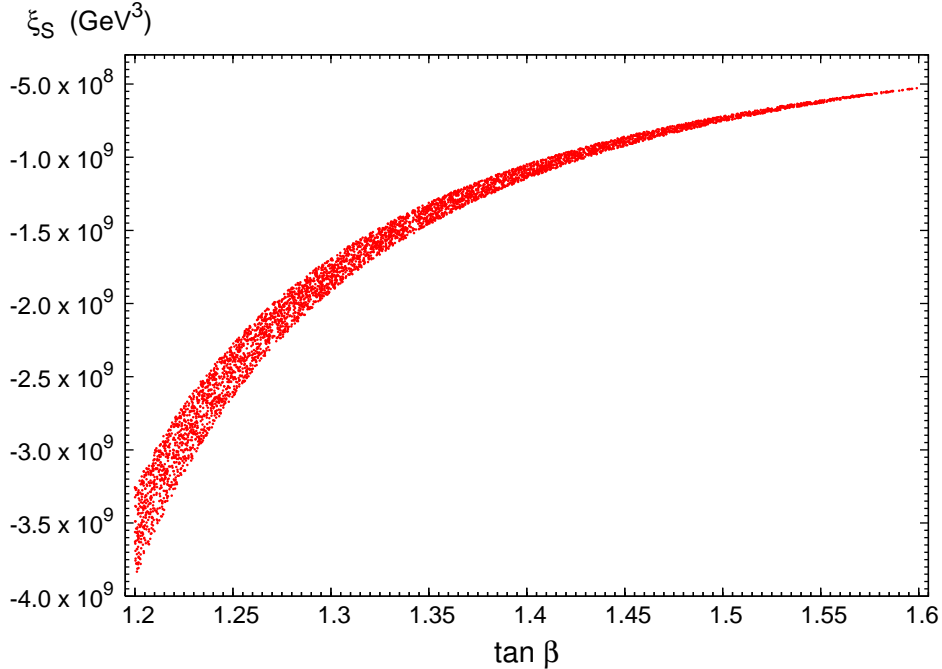
Figure 5: ξ_S as a function of $\tan \beta$ for the same parameters as in Fig. 2.

~ 5 TeV at M (still $\gtrsim 2$ TeV at the weak scale), which we interpret as a certain amount of fine tuning between the remaining parameters of the Higgs potential. We will not consider the region II below. Note that, as in [18], we obtain κ as an output (from the minimization equations of the Higgs potential with M_Z as input), which can hide the fine tuning required.

Limiting ourselves to $\Delta_H \lesssim 1.5$ ($|A_\kappa| \lesssim 5$ TeV), we were able to confirm the region I. In Table 1 we show the Higgs spectrum, and in Table 2 the essential features of the corresponding sparticle spectrum for a representative point P1 in region I, where $A_\kappa = -160$ GeV, $\Delta_H = 0.1$, $\lambda = 0.02$ and $\tan \beta = 6.6$ (leading to $m_S^2 \sim -2.8 \times 10^5$ GeV^2 in agreement with (3.31)). The point P2 in Tables 1 and 2 is in the region III of [18]: there one has $A_\kappa = -4.77$ TeV, $\Delta_H = 1.46$, $\lambda = 0.5$ and $\tan \beta = 1.64$ ($m_S^2 \sim -5.3 \times 10^6$ GeV^2). We see that, in spite of stop masses in the 2 TeV region, m_{h_1} is not far above the LEP bound. On the other hand these results confirm the phenomenological viability of the scenario proposed in [16, 18]. (However, due to the very heavy sparticle spectrum the supersymmetric contribution to the muon anomalous magnetic moment is still too small to account for the presently observed deviation w.r.t. the Standard Model.)

3.3 Scenarios without tadpole and A-terms

The scenario discussed in the previous subsection belongs to those where many (actually most) of the operators (2.7) – (2.12) and (2.22) – (2.23) are forbidden by a discrete Z_N symmetry, which is left unbroken in the messenger/sequestered sector, but under which $\widehat{S}$ carries a non-vanishing charge. In the above case – where Z_N is *not* an R-symmetry – all

Table 1: Input parameters and Higgs masses for five specific points.

Point	P1	P2	P3	P4	P5
Input parameters					
Messenger scale M (GeV)	10^{13}	10^{13}	4×10^{8}	3×10^{7}	5×10^{14}
$M_{SUSY} = m^2/M$ (GeV)	1.72×10^{5}	1.72×10^{5}	3.2×10^{4}	3.5×10^{4}	7.5×10^{4}
$\tan \beta$	6.6	1.64	1.6	1.9	40
n_5	2	2	2	2	2
λ	0.02	0.5	0.6	0.6	0.01
A_κ (GeV)	-160	-4770	0	0	0
Δ_H	0.1	1.46	0	0	0
m_S^2 (GeV^2)	-2.8×10^{5}	-5.3×10^{6}	-4.3×10^{5}	-2.1×10^{5}	-5.0×10^{3}
CP even Higgs masses					
$m_{h_1^0}$ (GeV)	116.1	115.8	115.5	96.1	94.5
$m_{h_2^0}$ (GeV)	794	2830	607	514	120
$m_{h_3^0}$ (GeV)	1762	3411	717	579	603
CP odd Higgs masses					
$m_{a_1^0}$ (GeV)	448	2842	40.5	11.5	1.1
$m_{a_2^0}$ (GeV)	1761	3662	628	546	603
Charged Higgs mass					
$m_{h^\pm}$ (GeV)	1764	2862	619	535	613

Table 2: Some sparticle masses and components for the five specific points of Table 1. The chargino masses are close to the wino/higgsino-like neutralino masses, the right-handed/left-handed slepton masses close to the stau$_1$/stau$_2$ masses, and the remaining squark masses are of the order of the gluino mass.

Point	P1	P2	P3	P4	P5
Neutralinos					
χ_1 mass (GeV)	467	469	80.5	88.3	101
Dominant component	bino	bino	bino	bino	singlino
χ_2 mass (GeV)	839	890	152	166	200
Dominant component	singlino	wino	wino	wino	bino
χ_3 mass (GeV)	882	2322	463	428	380
Dominant component	wino	higgsino	higgsino	higgsino	wino
χ_4 mass (GeV)	1432	2325	476	438	675
Dominant component	higgsino	higgsino	higgsino	higgsino	higgsino
χ_5 mass (GeV)	1440	4019	721	572	685
Dominant component	higgsino	singlino	singlino	singlino	higgsino
Stau$_1$ mass (GeV)	692	693	100	103	260
Stau$_2$ mass (GeV)	1100	1096	188	198	514
Stop$_1$ mass (GeV)	1931	1819	376	459	872
Gluino mass (GeV)	2389	2386	522	569	1117

soft terms m_S^2, $A_\kappa = 3A_\lambda$ and the parameter Δ_H in (2.24) will in general be non-vanishing (all others being forbidden).

The fate of R-symmetries in the context of gauge mediation has recently been reviewed in [25]. In the case of spontaneous breaking within the messenger/sequestered sector [26], R-symmetry violating terms in the effective low energy theory will be suppressed relative to R-symmetry conserving terms. Then, the trilinear terms $A_\kappa = 3A_\lambda$ (2.19) will be negligibly small. Although the R-symmetry breaking gaugino masses will typically also be smaller than the scalar masses at the messenger scale [25], we will consider in this subsection an illustrative scenario which is just a limiting case of the one previously discussed.

We will investigate the case where the trilinear terms vanish, and where only m_S^2 (which can never be forbidden by symmetries) assumes natural values at the messenger scale M. For simplicity, we will allow for standard gaugino masses (and the usual scalar masses) as given in appendix A. Now, the scalar sector of the NMSSM has an exact R-symmetry at the scale M, with identical charges for all superfields. Given that gaugino masses break this R-symmetry, radiative corrections (the RG running between M and the weak scale) induce R-symmetry violating trilinear terms in the scalar sector. If M is not too large or if λ, κ are small, these trilinear terms remain numerically small, and the R-symmetry in the scalar sector is only weakly broken. Given that this approximate R-symmetry is spontaneously broken at the weak scale by the VEVs of H_u, H_d and S, a pseudo Goldstone boson (a pseudo R-axion [27]) appears in the spectrum. Light pseudoscalars can lead to a reduction of the LEP constraints on m_{h_1}, and have recently been the subject of various investigations [28].

In what follows we study the phenomenological viability of such scenarios, which are defined by having all terms (2.16) – (2.21) vanish except for m_S^2 (but vanishing A_κ, A_λ). For simplicity we will also assume that Δ_H in (2.24) is negligibly small. Then, the model is completely specified by λ, $\tan\beta$ and the scales M and M_{SUSY} (recall that κ and m_S^2 can be obtained from the minimization equations in terms of M_Z and of the other parameters). Again we found that two completely different regions in parameter space are phenomenologically viable.

As before, the first region is characterized by small values of $\tan\beta$ ($\tan\beta \lesssim 2$) and large values of λ. Relatively large negative values for the soft mass m_S^2 for the singlet of the order $m_S^2 \sim -(600 \text{ GeV})^2$ are required at the scale M in order to generate the required VEV of S. The mass m_{a_1} of the lightest CP-odd scalar varies in the range $0 < m_{a_1} < 50$ GeV, where the larger values are obtained for larger messenger scales $M \sim 10^9$ GeV: then the RG evolution generates relatively large values $A_\lambda \sim 25$ GeV at the weak scale (whereas A_κ remains very small), and this breaking of the R-symmetry induces a relatively large mass for the pseudo R-axion. On the other hand, arbitrarily small values for A_λ and hence for m_{a_1} can be obtained without any fine tuning for lower messenger scales M. In all cases we find that the lightest CP-even (SM like) scalar h_1 dominantly decays (with branching ratios of $\sim 80\%$) into $h_1 \to a_1 a_1$, which allows for $m_{h_1} < 114$ GeV consistent with LEP constraints.

For given λ, m_{h_1} is nearly independent of the scales M and M_{SUSY}, but decreases with $\tan\beta$. In Fig. 6 we show a scatterplot for m_{h_1} as a function of $\tan\beta$, which is obtained for $\lambda = 0.6$, varying M in the range 10^7 GeV $< M < 5 \times 10^9$ GeV and M_{SUSY} in the range 3.3×10^4 GeV $< M_{SUSY} < 4.3 \times 10^4$ GeV. All points displayed satisfy LEP and

15

B-physics constraints. (We have chosen $n_5 = 2$ messenger multiplets, but similar results can be obtained – for slightly different ranges of M and M_{SUSY} – for $n_5 = 1$.)
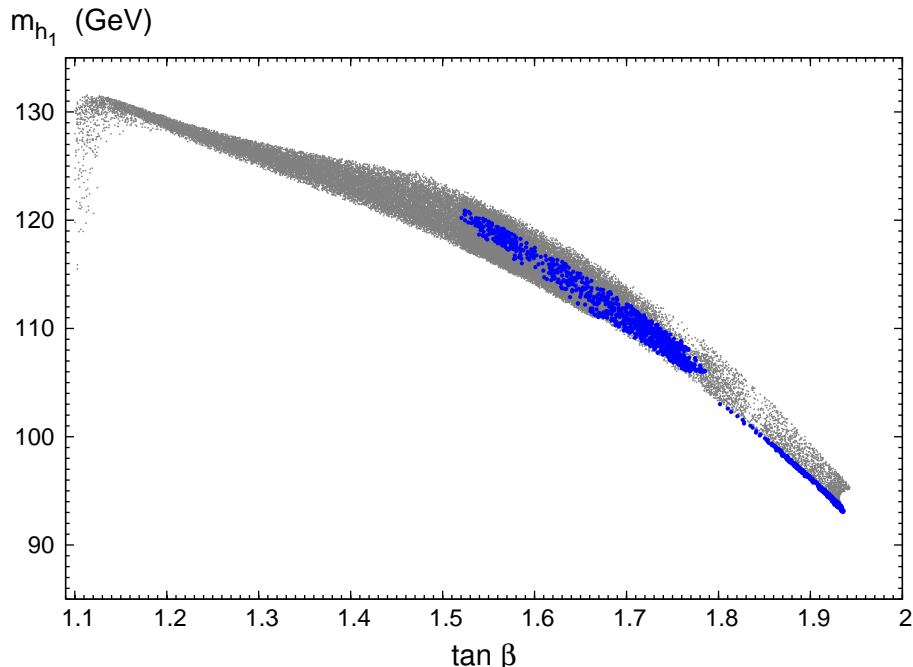

Figure 6: m_{h_1} as a function of $\tan \beta$ for $\lambda = 0.6$, 10^7 GeV $< M < 5 \times 10^9$ GeV and 3.3×10^4 GeV $< M_{SUSY} < 4.3 \times 10^4$ GeV. Points where, in addition to all LEP and B-physics constraints, the SUSY contribution to the muon anomalous magnetic moment can (fails to) account for the presently observed deviation with respect to the Standard Model are denoted in blue/darker (gray/lighter) color.

In the region $\tan \beta \gtrsim 1.7$ (where $m_{h_1} \lesssim 108$ GeV) LEP constraints are satisfied only for $m_{a_1} \lesssim 11$ GeV, so that $a_1 \to bb$ decays are forbidden and the dominant decays of h_1 are $h_1 \to a_1 a_1 \to 4\,\tau$ (still requiring $m_{h_1} \gtrsim 88$ GeV [22]). For $\tan \beta \lesssim 1.7$, the dominant decays of h_1 are $h_1 \to a_1 a_1 \to 4\,b$, in which case LEP constraints allow for m_{h_1} as low as ~ 108 GeV. The complete theoretically possible range for m_{a_1} is now allowed by LEP. (Fixing, e.g., $M = 10^8$ GeV, the complete range $1.2 \lesssim \tan \beta \lesssim 1.7$ is compatible with LEP constraints on the Higgs sector within the above range of M_{SUSY}. For smaller $\tan \beta$, however, the hidden fine tuning becomes quite large.)

Now, in some regions in parameter space, the supersymmetric contribution to the muon anomalous magnetic moment is $\gtrsim 10^{-9}$, which accounts for the presently observed deviation with respect to the Standard Model. The blue (darker) points in Fig. 6 (which appear only for $\tan \beta \gtrsim 1.5$) satisfy this condition. In Tables 1 and 2 we present the Higgs and sparticle spectrum for points P3 (with $\tan \beta = 1.6$) and P4 (with $\tan \beta = 1.9$), which are inside the blue region of Fig. 6.

Another interesting region in parameter space is characterized by large values of $\tan \beta$ ($\tan \beta \gtrsim 30$) and small values of λ ($\lambda \sim 10^{-2}$), associated with small values of κ ($\kappa \lesssim 10^{-3}$).

In this case, comparatively small negative values for the soft mass m_S^2 for the singlet of the order $m_S^2 \sim -(70 \text{ GeV})^2$ are required to generate the required VEV of S. Due to the small values of λ and κ, A_λ and A_κ remain small after the RG evolution from M down to the weak scale, leading to a pseudo R-axion with a mass $m_{a_1} \lesssim 1$ GeV. Now a_1 is particularly light since, for small κ, it simultaneously plays the rôle of a Peccei-Quinn pseudo Goldstone boson. However, due to the small value of λ, the couplings of a_1 (with doublet components $\lesssim 10^{-3}$) are tiny, and this CP-odd scalar would be very hard to detect; the branching ratios $h_i \to a_1 a_1$ are practically vanishing.

The CP-even Higgs sector is still compatible with LEP constraints if M is very large (and M_{SUSY} somewhat larger than above), leading to a sparticle spectrum (and A_t) in the 1 TeV range such that top/stop induced radiative corrections lift up the CP-even Higgs masses. Interestingly, in spite of $\lambda \sim 10^{-2}$, large values for $\mu_{eff} = \lambda \langle S \rangle$ still generate a large singlet/doublet mixing for the two lightest CP-even scalars. As an example, point P5 (which gives a satisfactory supersymmetric contribution to the muon anomalous magnetic moment) is shown in Tables 1 and 2. $m_{h_1} \sim 94$ GeV is well below 114 GeV, but the singlet component of h_1 is $\sim 88\%$ implying reduced couplings to gauge bosons. The state h_2 with a mass $m_{h_2} \sim 120$ GeV has still a singlet component $\sim 48\%$. With the help of its nonsinglet components, the detection of both states seems feasible at the ILC [29]. Also, the lightest neutralino is a nearly pure singlino (with nonsinglet components $\lesssim 3 \times 10^{-3}$), which would appear at the end of sparticle decay cascades [30].

Throughout this paper we have not addressed the issue of dark matter. Clearly, within GMSB models the LSP is the gravitino, but heavy remnants from the messenger sector can also contribute to the relic density [31, 32]. Its evaluation would require assumptions on the messenger/sequestered sector and the reheating temperature after inflation, and is beyond the scope of the present work. On the other hand, general considerations can possibly help to constrain the large variety of different scenarios found here.

4   Conclusions

We have seen in this analysis that the NMSSM can solve the μ-problem in GMSB models in a phenomenologically acceptable way. Our starting point was a derivation of the magnitude of all possible supersymmetric and soft terms in a generalized NMSSM, that can be radiatively generated by integrating out a sequestered/messenger sector with couplings to the singlet superfield $\widehat{S}$. For the phenomenological analysis, we confined ourselves to scenarios where most of these terms are negligibly small. Nevertheless we found a large variety of very different viable scenarios.

Scenarios with singlet tadpole terms *are* acceptable, if the linear terms in $\widehat{S}$ (or S) are generated to higher loop order only, or if at least one small Yukawa coupling is involved. A simple concrete model [4] with a direct coupling of $\widehat{S}$ to the messengers is viable for a Yukawa coupling $\eta \lesssim 10^{-5}$. In the case of models with forbidden tadpole terms, as those proposed in [16] and analysed in [18], we confirmed the phenomenological viability observed

in [18] (at least for the regions in parameter space without uncomfortably large values of A_κ).

Quite interesting from the phenomenological point of view are the scenarios with vanishing A-terms at the messenger scale: these automatically lead to a light CP-odd Higgs scalar as studied in [27, 28], which plays the rôle of a pseudo R-axion. In view of the simplicity with which these scenarios can satisfy LEP constraints, it would be very desirable to develop concrete models which generate this structure for the effective NMSSM at the scale M.

Finally we recall that the Fortran routine NMGMSB, that allowed to obtain the results above, will be available on the website [21]. With the help of corresponding input and output files, further properties of the points P1 to P5 as sparticle masses, couplings and branching ratios can be obtained.

Note added

After the completion of this paper another viable scenario was proposed in [34], in which the singlet does not couple to the messenger/sequestered sector, but where the source of supersymmetry breaking in the messenger sector is not SU(5) invariant.

Acknowledgements

Appendix A

In this appendix we summarize the expressions for the gaugino and scalar masses (at the scale M), which are generated by gauge mediation under the assumptions that the messenger sector involves n_5 $(5 + \bar{5})$ representations under $SU(5)$ (additional $(10 + \overline{10})$ representations can be taken care of by adding three units to n_5) with a common SUSY mass M, and F-type mass splittings m^2 among the scalars and pseudoscalars. The $U(1)_Y$ coupling α_1 is defined in the SM normalization (not in the GUT normalization). For convenience we define the scale $M_{SUSY} = m^2/M$ and the parameter $x = M_{SUSY}/M$ (typically $\ll 1$). The required one loop and two loop functions are [31, 33]

$$
\begin{aligned}
f_1(x) &= \frac{1}{x^2}\left((1+x)\ln(1+x) + (1-x)\ln(1-x)\right), \\
f_2(x) &= \frac{1+x}{x^2}\left(\ln(1+x) - 2Li_2\left(\frac{x}{1+x}\right) + \frac{1}{2}Li_2\left(\frac{2x}{1+x}\right)\right) + (x \to -x) ,
\end{aligned}
$$

which satisfy $f_1(x \to 0) = f_2(x \to 0) = 1$.

Then the gaugino masses are given by

$$
\begin{aligned}
M_1 &= \frac{\alpha_1}{4\pi} M_{SUSY} f_1(x) \frac{5}{3} n_5 \;, \\
M_2 &= \frac{\alpha_2}{4\pi} M_{SUSY} f_1(x) n_5 \;, \\
M_3 &= \frac{\alpha_3}{4\pi} M_{SUSY} f_1(x) n_5 \;,
\end{aligned}
$$

and the scalar masses squared by

$$
m^2 \;=\; \frac{M_{SUSY}^2}{16\pi^2} \left(\frac{10}{3} Y^2 \alpha_1^2 + \frac{3}{2} \alpha_2^{2(1)} + \frac{8}{3} \alpha_3^{2(2)} \right) f_2(x) n_5 \;.
$$

The terms [(1)] are present for $SU(2)$ doublets only, and the terms [(2)] for $SU(3)$ triplets only. The hypercharges Y are

	u_L/d_L	u_R	d_R	ν_L/e_L	e_R	H_u, H_d
Y	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	$\pm\frac{1}{2}$

Appendix B

In this appendix we give the one loop β-functions for the parameters in the Higgs sector of the general NMSSM, defined by a superpotential

$$
\begin{aligned}
W \;=\; & \lambda \widehat{S}\widehat{H}_u\widehat{H}_d + \frac{\kappa}{3}\widehat{S}^3 + \mu \widehat{H}_u\widehat{H}_d + \mu'\widehat{S}^2 + \xi_F\widehat{S} \\
& + h_t \widehat{Q}_3\widehat{H}_u\widehat{T}_R^c - h_b \widehat{Q}_3\widehat{H}_d\widehat{B}_R^c - h_\tau \widehat{L}_3\widehat{H}_d\widehat{L}_R^c
\end{aligned}
$$

and soft terms

$$
\begin{aligned}
V_{soft} \;=\; & m_u^2 |H_u|^2 + m_d^2 |H_d|^2 + m_S^2 |S|^2 + (\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 + m_3^2 H_u H_d + m_s'^2 S^2 + \xi_S S \\
& + h_t A_t Q_3 H_u T_R^c - h_b A_b Q_3 H_d B_R^c - h_\tau A_\tau L_3 H_d L_R^c + h.c.) \;,
\end{aligned}
$$

under the assumption $\sum_i Y_i m_i^2 = 0$, which is always satisfied for GMSB models.

$$
\begin{aligned}
\frac{d\lambda^2}{d\ln Q^2} &= \frac{\lambda^2}{16\pi^2} \left(4\lambda^2 + 2\kappa^2 + 3(h_t^2 + h_b^2) + h_\tau^2 - g_1^2 - 3g_2^2 \right) \\
\frac{d\kappa^2}{d\ln Q^2} &= \frac{\kappa^2}{16\pi^2} \left(6\lambda^2 + 6\kappa^2 \right) \\
\frac{dh_t^2}{d\ln Q^2} &= \frac{h_t^2}{16\pi^2} \left(\lambda^2 + 6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 \right)
\end{aligned}
$$

19

$$\frac{dh_b^2}{d\ln Q^2} = \frac{h_b^2}{16\pi^2}\left(\lambda^2 + 6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{9}g_1^2\right)$$

$$\frac{dh_\tau^2}{d\ln Q^2} = \frac{h_\tau^2}{16\pi^2}\left(\lambda^2 + 3h_b^2 + 4h_\tau^2 - 3g_2^2 - 3g_1^2\right)$$

$$\frac{d\mu}{d\ln Q^2} = \frac{\mu}{16\pi^2}\left(\lambda^2 + \frac{3}{2}(h_t^2 + h_b^2) + \frac{1}{2}h_\tau^2 - \frac{1}{2}(g_1^2 + 3g_2^2)\right)$$

$$\frac{d\mu'}{d\ln Q^2} = \frac{\mu'}{16\pi^2}\left(2\lambda^2 + 2\kappa^2\right)$$

$$\frac{d\xi_F}{d\ln Q^2} = \frac{\xi_F}{16\pi^2}\left(\lambda^2 + \kappa^2\right)$$

$$\frac{dA_\lambda}{d\ln Q^2} = \frac{1}{16\pi^2}\left(4\lambda^2 A_\lambda + 2\kappa^2 A_\kappa + 3(h_t^2 A_t + h_b^2 A_b) + h_\tau^2 A_\tau + g_1^2 M_1 + 3g_2^2 M_2\right)$$

$$\frac{dA_\kappa}{d\ln Q^2} = \frac{1}{16\pi^2}\left(6\lambda^2 A_\lambda + 6\kappa^2 A_\kappa\right)$$

$$\frac{dA_t}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\lambda^2 A_\lambda + 6h_t^2 A_t + h_b^2 A_b + \frac{13}{9}g_1^2 M_1 + 3g_2^2 M_2 + \frac{16}{3}g_3^2 M_3\right)$$

$$\frac{dA_b}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\lambda^2 A_\lambda + 6h_b^2 A_b + h_t^2 A_t + h_\tau^2 A_\tau + \frac{7}{9}g_1^2 M_1 + 3g_2^2 M_2 + \frac{16}{3}g_3^2 M_3\right)$$

$$\frac{dA_\tau}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\lambda^2 A_\lambda + 3h_b^2 A_b + 4h_\tau^2 A_\tau + 3g_1^2 M_1 + 3g_2^2 M_2\right)$$

$$\frac{dm_u^2}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\lambda^2(m_u^2 + m_d^2 + m_S^2 + A_\lambda^2) + 3h_t^2(m_u^2 + m_T^2 + m_Q^2 + A_t^2) + \frac{g_1^2}{2}(m_u^2 - m_d^2)\right.$$
$$\left. - g_1^2 M_1^2 - 3g_2^2 M_2^2\right)$$

$$\frac{dm_d^2}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\lambda^2(m_u^2 + m_d^2 + m_S^2 + A_\lambda^2) + 3h_b^2(m_d^2 + m_B^2 + m_Q^2 + A_b^2)\right.$$
$$\left. + h_\tau^2(m_d^2 + m_\tau^2 + m_L^2 + A_\tau^2) - \frac{g_1^2}{2}(m_u^2 - m_d^2) - g_1^2 M_1^2 - 3g_2^2 M_2^2\right)$$

$$\frac{dm_S^2}{d\ln Q^2} = \frac{1}{16\pi^2}\left(2\lambda^2(m_u^2 + m_d^2 + m_S^2 + A_\lambda^2) + \kappa^2(6m_S^2 + 2A_\kappa^2)\right)$$

$$\frac{dm_3^2}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\frac{m_3^2}{2}(6\lambda^2 + 3h_t^2 + 3h_b^2 + h_\tau^2 - g_1^2 - 3g_2^2) + 2\lambda\kappa m_S'^2\right.$$
$$\left. + \mu(2\lambda^2 A_\lambda - 3h_t^2 A_t - 3h_b^2 A_b - h_\tau^2 A_\tau + g_1^2 M_1 + 3g_2^2 M_2)\right)$$

$$\frac{dm_S'^2}{d\ln Q^2} = \frac{1}{16\pi^2}\left(m_S'^2(2\lambda^2 + 4\kappa^2) + 2\lambda\kappa m_3^2 + 4\mu'(\lambda^2 A_\lambda + \kappa^2 A_\kappa)\right)$$

$$\frac{d\xi_S}{d\ln Q^2} = \frac{1}{16\pi^2}\left(\xi_S(\lambda^2 + \kappa^2) + 2\lambda\mu(m_u^2 + m_d^2) + 4\kappa\mu' m_S^2 + 2\lambda m_3^2(A_\lambda + 2\mu')\right.$$
$$\left. + 2\xi_F(\lambda^2 A_\lambda + \kappa^2 A_\kappa + 2\kappa m_S'^2(A_\kappa + 2\mu'))\right)$$

References

[1] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B **189** (1981) 575;
S. Dimopoulos and S. Raby, Nucl. Phys. B **192** (1981) 353;
M. Dine and M. Srednicki, Nucl. Phys. B **202** (1982) 238.

[2] M. Dine and W. Fischler, Phys. Lett. B **110** (1982) 227;
M. Dine and W. Fischler, Nucl. Phys. B **204** (1982) 346;
L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B **207** (1982) 96;
S. Dimopoulos and S. Raby, Nucl. Phys. B **219** (1983) 479.

[3] N. Dragon, U. Ellwanger and M. G. Schmidt, Phys. Lett. B **145** (1984) 192; Nucl. Phys. B **255** (1985) 549; Phys. Lett. B **154** (1985) 373; Z. Phys. C **29** (1985) 209.

[4] U. Ellwanger, Phys. Lett. B **349** (1995) 57 [arXiv:hep-ph/9501227].

[5] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **241** (1984) 493; Phys. Lett. B **137** (1984) 187; Phys. Lett. B **140** (1984) 59; Nucl. Phys. B **256** (1985) 557;
Y. Meurice and G. Veneziano, Phys. Lett. B **141** (1984) 69.

[6] M. Dine and A. E. Nelson, Phys. Rev. D **48** (1993) 1277 [arXiv:hep-ph/9303230];
M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D **51** (1995) 1362 [arXiv:hep-ph/9408384];
M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53** (1996) 2658 [arXiv:hep-ph/9507378].

[7] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [arXiv:hep-th/0602239].

[8] P. Fayet, Nucl. Phys. B **90** (1975) 104; Phys. Lett. B **64** (1976) 159; Phys. Lett. B **69** (1977) 489 and Phys. Lett. B **84** (1979) 416.

[9] H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120** (1983) 346;
J.M. Frere, D.R. Jones and S. Raby, Nucl. Phys. B **222** (1983) 11.

[10] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.

[11] A. Hebecker, J. March-Russell and R. Ziegler, "Inducing the μ and the $B\mu$ Term by the Radion and the 5d Chern-Simons Term," arXiv:0801.4101 [hep-ph].

[12] J. Polchinski and L. Susskind, Phys. Rev. D **26** (1982) 3661;
H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **124** (1983) 337;
A. B. Lahanas, Phys. Lett. B **124** (1983) 341;
U. Ellwanger, Phys. Lett. B **133** (1983) 187;
J. Bagger and E. Poppitz, Phys. Rev. Lett. **71** (1993) 2380 [arXiv:hep-ph/9307317];
J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B **455** (1995) 59 [arXiv:hep-ph/9505244].

[13] D. Nemeschansky, Nucl. Phys. B **234** (1984) 379.

[14] G. R. Dvali, G. F. Giudice and A. Pomarol, Nucl. Phys. B **478** (1996) 31 [arXiv:hep-ph/9603238].

[15] P. Ciafaloni and A. Pomarol, Phys. Lett. B **404** (1997) 83 [arXiv:hep-ph/9702410]; K. Agashe and M. Graesser, Nucl. Phys. B **507** (1997) 3 [arXiv:hep-ph/9704206].

[16] G. F. Giudice and R. Rattazzi, Nucl. Phys. B **511** (1998) 25 [arXiv:hep-ph/9706540].

[17] A. de Gouvea, A. Friedland and H. Murayama, Phys. Rev. D **57** (1998) 5676 [arXiv:hep-ph/9711264]; T. Han, D. Marfatia and R. J. Zhang, Phys. Rev. D **61** (2000) 013007 [arXiv:hep-ph/9906508]; Z. Chacko and E. Ponton, Phys. Rev. D **66** (2002) 095004 [arXiv:hep-ph/0112190]; M. Dine and J. Mason, "Dynamical Supersymmetry Breaking and Low Energy Gauge Mediation," arXiv:0712.1355 [hep-ph].

[18] A. Delgado, G. F. Giudice and P. Slavich, Phys. Lett. B **653** (2007) 424 [arXiv:0706.3873 [hep-ph]].

[19] G. F. Giudice, H. D. Kim and R. Rattazzi, "Natural mu and Bmu in gauge mediation," arXiv:0711.4448 [hep-ph].

[20] B. C. Allanach *et al.*, "SUSY Les Houches Accord 2," arXiv:0801.0045 [hep-ph].

[21] U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP **0502** (2005) 066 [arXiv:hep-ph/0406215]; U. Ellwanger and C. Hugonie, Comput. Phys. Commun. **177** (2007) 399, http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html

[22] S. Schael *et al.* [ALEPH, DELPHI, L3 and OPAL Collaborations], Eur. Phys. J. C **47** (2006) 547

[23] F. Domingo and U. Ellwanger, JHEP **0712** (2007) 090 [arXiv:0710.3714 [hep-ph]].

[24] C. Panagiotakopoulos and A. Pilaftsis, Phys. Rev. D **63** (2001) 055003 [arXiv:hep-ph/0008268]; A. Dedes, C. Hugonie, S. Moretti and K. Tamvakis, Phys. Rev. D **63** (2001) 055009 [arXiv:hep-ph/0009125]; A. Menon, D. E. Morrissey and C. E. M. Wagner, Phys. Rev. D **70** (2004) 035005 [arXiv:hep-ph/0404184].

[25] S. A. Abel, C. Durnford, J. Jaeckel and V. V. Khoze, "Patterns of Gauge Mediation in Metastable SUSY Breaking," arXiv:0712.1812 [hep-ph].

[26] S. Abel, C. Durnford, J. Jaeckel and V. V. Khoze, "Dynamical breaking of $U(1)_R$ and supersymmetry in a metastable vacuum," arXiv:0707.2958 [hep-ph].

[27] B. A. Dobrescu, G. Landsberg and K. T. Matchev, Phys. Rev. D **63** (2001) 075003 [arXiv:hep-ph/0005308];
B. A. Dobrescu and K. T. Matchev, JHEP **0009** (2000) 031 [arXiv:hep-ph/0008192];
R. Dermisek and J. F. Gunion, Phys. Rev. D **75** (2007) 075019 [arXiv:hep-ph/0611142].

[28] R. Dermisek and J. F. Gunion, Phys. Rev. Lett. **95** (2005) 041801 [arXiv:hep-ph/0502105] and Phys. Rev. D **73** (2006) 111701 [arXiv:hep-ph/0510322];
U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP **0507** (2005) 041 [arXiv:hep-ph/0503203];
S. Chang, P. J. Fox and N. Weiner, JHEP **0608** (2006) 068 [arXiv:hep-ph/0511250];
P. W. Graham, A. Pierce and J. G. Wacker, "Four taus at the Tevatron," arXiv:hep-ph/0605162;
S. Moretti, S. Munir and P. Poulose, Phys. Lett. B **644** (2007) 241 [arXiv:hep-ph/0608233];
S. Chang, P. J. Fox and N. Weiner, Phys. Rev. Lett. **98** (2007) 111802 [arXiv:hep-ph/0608310];
T. Stelzer, S. Wiesenfeldt and S. Willenbrock, Phys. Rev. D **75** (2007) 077701 [arXiv:hep-ph/0611242];
U. Aglietti *et al.*, "Tevatron-for-LHC report: Higgs," arXiv:hep-ph/0612172;
E. Fullana and M. A. Sanchis-Lozano, Phys. Lett. B **653** (2007) 67 [arXiv:hep-ph/0702190];
K. Cheung, J. Song and Q. S. Yan, Phys. Rev. Lett. **99** (2007) 031801 [arXiv:hep-ph/0703149];
M. A. Sanchis-Lozano, "A light non-standard Higgs boson: to be or not to be at a (Super) B factory?," arXiv:0709.3647 [hep-ph];
M. Carena, T. Han, G. Y. Huang and C. E. M. Wagner, arXiv:0712.2466 [hep-ph];
J.R. Forshaw, J.F. Gunion, L. Hodgkinson, A. Papaefstathiou and A.D. Pilkington, arXiv:0712.3510 [hep-ph];
Z. Heng, R. J. Oakes, W. Wang, Z. Xiong and J. M. Yang, "B meson Dileptonic Decays in NMSSM with a Light CP-odd Higgs Boson," arXiv:0801.1169 [hep-ph];
A. Djouadi *et al.*, "Benchmark scenarios for the NMSSM," arXiv:0801.4321 [hep-ph].

[29] U. Ellwanger, J. F. Gunion, C. Hugonie and S. Moretti, "Towards a no-lose theorem for NMSSM Higgs discovery at the LHC," arXiv:hep-ph/0305109, published in G. Weiglein *et al.* [LHC/LC Study Group], Phys. Rept. **426** (2006) 47 [arXiv:hep-ph/0410364].

[30] U. Ellwanger and C. Hugonie, Eur. Phys. J. C **5** (1998) 723 [arXiv:hep-ph/9712300] and Eur. Phys. J. C **13** (2000) 681 [arXiv:hep-ph/9812427];
V. Barger, P. Langacker and G. Shaughnessy, Phys. Lett. B **644** (2007) 361 [arXiv:hep-ph/0609068].

[31] S. Dimopoulos, G. F. Giudice and A. Pomarol, Phys. Lett. B **389**, 37 (1996) [arXiv:hep-ph/9607225].

[32] T. Han and R. Hempfling, Phys. Lett. B **415** (1997) 161 [arXiv:hep-ph/9708264];
K. Jedamzik, M. Lemoine and G. Moultaka, Phys. Lett. B **645** (2007) 222
[arXiv:hep-ph/0504021] and Phys. Rev. D **73** (2006) 043514 [arXiv:hep-ph/0506129].

[33] S. P. Martin, Phys. Rev. D **55** (1997) 3177 [arXiv:hep-ph/9608224].

[34] T. Liu and C. E. M. Wagner, "Dynamically Solving the μ/B_μ Problem in Gauge-mediated Supersymmetry Breaking," arXiv:0803.2895 [hep-ph].